A Theory of Efficient Public-Private Capital Structures

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Abstract

This paper presents a model to assess the efficiency of the capital structure in public-private partnerships (PPP). A main argument supporting the PPP approach for investment projects is the transfer of know-how from the private partner to the investment vehicle. The paper shows how different knowledge transfer schemes determine an optimal shareholding structure of the PPP. Under the assumption of lower capital cost of the public partner and lower development outlays when the investment is carried out by a private investor, an optimal capital structure is achieved with both the public and the private parties as shareholders.

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1. Introduction

The provision of public goods provides a strong rationale for public-private hybrids that can efficiently carry out public investment projects. With the present financial crisis, the debate on the role of public-private partnerships (PPPs) has attracted new attention. The key justifications for pursuing PPPs as quoted in the literature are lower costs than in sole private investments and higher quality than in the sole public provision of the public good.\(^1\) The lower costs of PPPs result from lower cost of capital of the public partner, while higher quality is achieved due to the transfer of know-how from the private to the public partner.

In the case of PPP projects in infrastructure, empirical research carried out mostly in the U.K. and the U.S. shows that the private sector is indeed able to build infrastructure cheaper than the public sector.\(^2\) The savings amount to 15-30 percent\(^3\) and can be attributed to more efficient project management by the private investor, shorter construction time, as well as lower administrative expenses. On the other hand, in developed markets the cost of capital for the private sector is on average 40-260 basis points higher than for the public sector.\(^4\)

PPPs are often undertaken by joint venture companies (special purpose vehicles, SPV) with equity contributed by the private and public sectors.\(^5\) The share in equity of the SPV is usually reflected in the voting power of the partners. To be attractive and viable, the shareholding structure of the SPV should secure the interest of both the public and private partner, namely providing enough public capital and sufficient private sector know-how. Efficient financing of public investments by the private sector also requires that the higher financing cost of the private sector must be offset by the savings of development outlays due to the transfer of know-how from the private sector. Some authors do not acknowledge a continuum between sole public or sole private shareholding for determining the optimal capital structure.\(^6\) The models presented below will show that the project may be optimally financed with the mixed public-private shareholding.

The literature on financing public investments by private capital compares higher outlays on construction for the public sector with the higher payments for a private investor.\(^7\) However, very few authors take account of the continuum between these two cost drivers and implications for the capital structure.\(^8\)

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2 See Wright (1987) and Viscusi et al. (2000).
3 Wallance and Junk (1970, quotation from Viscusi et al., 2000) even claim that public enterprises have investment outlays 40 percent higher than private ones.
4 See American Chamber of Commerce in Poland (2002) and Appendix.
6 For example American Chamber of Commerce in Poland (2002), Grout (1997), and Grout (2003).
7 Irwin (2008), p. 109. For the so-called Public Sector Comparator (PSC), see e.g., Broadbent and Laughlin (2003); UK National Audit Office: www.nao.org.uk/guidance/focus/000154_pp5-6.pdf.
This paper analyzes the conditions for PPPs which will provide for a Pareto-efficient development of public utilities infrastructure. Pareto optimality refers to achieving the same quality of service at a lower cost or better quality at the same cost for publicly developed or privately developed public investments. It will also evaluate how different forms of knowledge transfer determine an optimal capital structure of the SPV. I find that if the cost of capital is lower for public entities and the outlays on building infrastructure are lower when the investment is made by a private investor, it is possible to reach the lowest total cost of construction with both public and private capital as part of the shareholding.

The paper is organized as follows. Section 2 presents the justification for the key assumptions of the model about the lower cost of capital than the private partner and the assumption that the private sector can construct the infrastructure at the lower cost than the public sector. Section 3 lays out the relationships between the level of quality, the required investments in infrastructure, and the cost of public utilities’ services. Section 4 presents a model of knowledge transfer (initially a discrete model, then a continuous one) to describe the optimal public-private capital structure in PPPs for the provision of public utilities infrastructure. It also includes practical examples of existing PPPs in the public utilities sector. Section 5 concludes with policy advice on the applicability of the model solution.

2. Cost of capital and savings in Public-Private Partnerships

The underlying assumption of the paper is that it might be optimal for the public partner to become a shareholder in the SPV. In the markets with a long and reliable tradition of public projects procurement to the private sector such an assumption might be counterintuitive. However, the existence of market failures, especially in the case of transition and emerging economies, justifies the government’s involvement in the SPV to correct market failures which result in the inability of the marketplace to provide public goods.

One of the market failures relates to the financial market. The domestic financial markets of emerging economies are not deep and efficient enough to provide reasonably priced capital to finance projects realized for the public. Therefore governments aiming at development of the infrastructure have to use the savings of the external sector. The current account deficits of the Central and Eastern European countries confirm this observation. Therefore the participation of the public sector in the projects provides a necessary certainty to the financial viability of the project. It also serves as the necessary condition to attract the foreign capital to participate in financing of the project. Of course, the public participation does not correct all of the market failures but at least may lead to the reduction of their impact. On the other hand, the conservative approach to risk evaluation by the foreign investors leads to adding additional premium to cover an excessive risk which is difficult to evaluate. The additional premium – an extra reward for uncertainty – may be the reason of a market failure of public investment projects. The public sector’s participation may offset this additional premium and make public investment an interesting business case.

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9 In fact, when we presented the paper for the first time in mid-2007 for review, well ahead the economic crisis, we got several remarks questioning the rationale of governments by taking a stake in private companies.

The reason behind a public shareholding in the SPV may be also explained from the point of view of the principal-agent theory. In a PPP, the public and private sector might have conflicting interests regarding the cost of the project and the quality of the public good delivered. Under most legislative systems, it is the shareholder who has much more control over the activities of the company than its contractor. Therefore, the government seeking a high quality and reasonably priced public good would have much more control over the project when it is a shareholder of the SPV. Again, the need for better control of the quality and prices may be more important in the emerging and transition markets, where public legal regulations are not entrenched strongly enough.

2.1. Cost of the public capital

Some researches develop a notion of the “social discount rate” that should be used to discount and compare the value for the public sector achieved in the public projects. Many academicians analyzing the social discount rate in the 1960’s and 1970’s claimed that the discount rate for public entities should be lower than for the private sector.\(^\text{11}\) Other researches claim that the social discount rate should be higher than the public borrowing cost.\(^\text{12}\) They suggest that the lower borrowing cost of the public sector reflects the fact that the public sector does not default or it can levy taxes to repay the debt and that the lower cost of fund does not reflect a more efficient management of risk.\(^\text{13}\) These comments may be true in the general case of public borrowing, especially in a closed economy. However, such arguments should not apply to the PPPs, especially those organized in emerging and transition markets, because of the following reasons.

- The effective PPP scheme assumes the transfer of risks from the public sector to the private sector (Blöndal, 2005). Therefore the public sector cost of borrowing may not reflect the project risks, as these risks are transferred out from the public sector. As the IMF (2004) paper states, it is difficult to establish whether the transfer of risk out of the public sector is proportional to the difference of funding cost, but the public cost of lending is definitely lower.

- In case of investments which are not very important to the economy (practically: low investments in relation to GDP), the discount rate for the public sector should be lower than for the private sector, because the public sector can better absorb and spread the risks among a greater number of people (Arrow and Lind, 1970; Fisher, 1973).

- Private companies cannot internalize the externalities, and their return on investment comes only from a given project. Flemming and Mayer (1997) showed that the private sector investments in the public utilities sector depend on the policy applied in other sectors of the economy and create externalities leading to inefficient piece-meal

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\(^\text{11}\) See Samuelson (1964); Vickrey (1964); Solow (1965); Baumol (1968); Arrow and Lind (1970).


\(^\text{13}\) We did not sustain that PPP projects become “less risky”. The risk profile of the project remains the same independently of the form of financing. In this paper, we do not tackle the issue of risk sharing which is essentially part of the operating phase.
decision making, when only a perspective of a single sector of the economy is taken into account.

- In case of incomplete capital markets, capital investors and lenders are not able to protect themselves in the capital market against the risk connected with securities that are financing the public or private investment and therefore apply various discount rates (Hirshleifer, 1964; Bailey and Jensen, 1972).
- Grout (2003) proved that even in the world without incomplete markets and distorting taxation, it is appropriate to apply a higher discount rate for private entities than for public ones. The argument is based on the differential of beta (risk) for government payments under a PPP and the beta of government expenditures under the normal public provision of goods.
- Lind (1990) suggests that the government’s long-term borrowing rate is a “good first candidate” for long-run intergenerational problems and that “for most government projects we should compute net benefits (from the project) using the government borrowing rate as the discount rate” (quote after Spackman, 2004).

Moreover, the concept of the social discount rate is based on the social time preference. It would be justified to take into account the social discount rate for the projects financed in the domestic market. However, it would be difficult to any government to consider the social discount rate of the external sector, which theoretically should be applied if the projects are financed from the savings of the external sector.

The two concepts: the cost of capital (i.e. a financial approach) and the social discount rate (social cost of capital, i.e. a general equilibrium economic approach) should not be equaled. I based my assumptions regarding the cost of capital on direct market evidence that the public sector can raise capital cheaper than the private sector\textsuperscript{14} and, from this pot of money, can finance different projects. I concentrated on the interests paid on the resources engaged and did not discuss the social cost of capital, which is a topic that has spun off considerable literature as seen above. Whether public sector projects should be discounted at a lower rate than private sector projects is a highly contentious issue and one that has spawned an enormous literature.

The amount of literature supporting the lower cost of capital for the public sector (starting from Arrow-Lind theorem in 1970) is at least so large and so strong as for those supporting the approach according to which public projects should be discounted at the same rate,\textsuperscript{15} independently of the source of financing.\textsuperscript{16} Writing in the 1980s on public sector discount rates and their relation to private sector discount rates, Lind (1982) pointed out that “the profession was no closer to agreement on the theory, on a procedure for computing the discount rate, or on the rate itself than it was in 1966”.

\textsuperscript{14} See Appendix. For Emerging Markets the bond market is not liquid enough to provide with continuous quotations. Most of municipal and corporate bonds are privately placed and have not unveiled yields. Moszoro and Kowalik (2005) listed a few municipal and corporate bonds with open yields formulas.\textsuperscript{15} See for example HM Treasury (2003b) and Engel, Fischer, Galetovic (2008).\textsuperscript{16} See the discussion in section 2.4 in Grout (2005).
2.2. Savings on construction cost due to the private sector’s participation in the SPV

The empirical research carried out mostly in the United Kingdom and in the United States shows that the private sector can construct the infrastructure cheaper than the public sector (Wright, 1987, Viscusi et al., 2000). Wallace and Junk (1970, quotation from: Viscusi et al., 2000, p. 448) even claim that public enterprises have investment costs 40 percent higher than the private ones.

On the one hand, PPPs involve sizeable structuring and transaction costs, and require governments to develop expensive internal capacities (lawyers, financial experts). However, on the other hand, savings can be attributed to the more efficient project management by the private investor, shortening of the time of construction and development, as well as lower administrative expenses (lower bureaucracy). Blöndal (2005) quotes an interesting statistics confirming the above mentioned sources of savings in the case of the United Kingdom (H.M. Treasury, 2003a):

- “Nearly 90 percent of all PPP projects were delivered on time by the private partner whereas only approximately 30 percent of non-PPP projects were delivered on time.
- Four-fifths of all PPP projects were delivered on budget whereas only one-fourth of non-PPP projects were delivered on budget. All PPP projects that experienced budget overruns were due to changes in requirements by the government.”

We did not sustain that the cost of construction is always higher for the public sector. I allow it to be higher and analyze what the consequences of this fact are. I based my assumption regarding possible higher costs of building by the public sector on market evidence.

The mainstream PPP literature suggests that the key issues involved are the bundling together of the construction and the operation of the project and the efficient allocation of risk between the public and private partners, regardless the capital structure. The bundling of contracts is not in contradiction to my assumption. It may even reinforce it. What the private sector does when bundling is a “package selling” of: construction, insurance, financing for lower price that when contracted independently by the public sector. I sustain that when the share of private capital is large enough, the advantages of this bundling may be realized because of the economic incentives for the private partner.

The private sector, when contracted by the public sector only for building (i.e. with no shares in the SPV according to my model) has no incentives to build cheaply but bidding low to win the tender. Where competition is low, it may happen (and unfortunately happens frequently) that:

- there is price collusion or ex-ante agreements who should win the tender,
- after bidding low and winning the tender, the private sector renegotiates the contracts.

When the private partner has a stake in the project, it has an economic incentive to transfer know-how to build cheaply.

3. The relationships within the model

The amount of capital expenditures on infrastructure is determined by the required quality of public service supplied by the SPV. Consider a newly formed SPV that has to invest

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during the development phase and – after the investment phase – operates the infrastructure and provides the public good. Further assume that during the operational phase of the project the public service is provided by the SPV at the cost that reflects a fixed fee which covers the amortization of the initial investment outlays and a variable fee that covers the current cost of service. This assumption requires that the capital expenditure \( I(q) \) required to satisfy the demand at the quality level \( q \) should be equal to the present value of fixed fees paid to the SPV \( f(q) \) over the life of the project \( t \):

\[
I(q) = f(q) \frac{1-(1+r)^{-t}}{r}
\]  

For a sufficiently long life of the project (for \( t \to \infty \)) this can be expressed as:

\[
I(q) = \frac{f(q)}{r}
\]  

This conclusion can be also derived from the market-clearing condition. In the two-part tariff system, for a given quality of service \( q \) and demand function \( P(x,q) \), the fee for consumption of \( x \) units of the service should include a fixed fee \( f(q) \) and a variable fee \( p(x,q) \).

\[
P(x,q) \cdot x = f(q) + p(x,q) \cdot x
\]  

From formula (3) the demand function is:

\[
P(x,q) = \frac{f(q)}{x} + p(x,q)
\]  

On the supply side, for each level of service provision \( x \), total revenues must include amortization of the investment outlays and a unit price at least equal to the marginal cost of service. This leads to the following supply function:

\[
S(x,q) = \frac{I(q) \cdot r}{x} + \text{MC}(x,q)
\]  

Market clearing condition requires \( P(x,q) \) for a given \( x \) and \( q \) to be equal to \( S(x,q) \)

\[
P(x,q) = S(x,q)
\]

\[
\frac{f(q)}{x} + p(x,q) = \frac{I(q) \cdot r}{x} + \text{MC}(x,q)
\]

\[
f(q) = I(q) \cdot r
\]  

4. **Determining the optimal public-private capital structure**

As stated above, it is assumed that the private sector is able to execute the PPP project cheaper than the public sector. Let us denote by \( J(q) \) the amount by which development
outlays (without financial costs) for a privately executed project are lower than the outlays for a publicly executed one.

Assume also that the transfer of the idiosyncratic assets of the private partner (such as know-how) to be transferred to the SPV materializes when the private share in the partnership’s capital achieves a minimum of \( e \). Then, as in equation (8), one may determine the level of fixed fees for mixed public-private financing from the following:

\[
f(q) = \theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot (I(q) + (1-\beta) \cdot J(q)) \cdot r_{pu}
\] (9)

where:

- \( r_{pr} \) – interest (discount) rate for a private investor,
- \( r_{pu} \) – interest (discount) rate for the public sector, \( r_{pr} > r_{pu} \) \(^{18}\),
- \( \theta \) – share of a private investor in the joint venture, \( \theta \in <0,1> \),
- \( \beta \) – discrete variable reflecting the existence of know-how in project execution, so that:

\[
\beta = \begin{cases} 
0 & \text{when there is no know-how transfer (} \theta < e) \\
1 & \text{when there is know-how transfer (} \theta \geq e) 
\end{cases}
\]

Thus the condition for a PPP to execute the investment at a lower cost than the public partner can be written as:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < (I(q) + (1-\beta) \cdot J(q)) \cdot r_{pu}
\] (10)

Sorting and arranging with regard to \( \theta \) one obtains:

\[
\theta \left( \frac{r_{pr}}{r_{pu}} - 1 \right) < \frac{(1-\beta) \cdot J(q)}{I(q)}
\]

or

\[
\theta < \frac{(1-\beta) \cdot J(q)}{I(q)} \left( \frac{r_{pr}}{r_{pu}} - 1 \right)
\] (11)

Condition (11) shows that the project should be fully realized by the public entity if either the private partner does not contribute by allowing savings related to its knowledge, i.e., \( J(q) = 0 \), or the savings are relatively small compared with the difference in financial costs.

Provided there is know-how transfer from the private to the public sector (i.e., \( \beta = 1 \)), a PPP (interior solution of inequality (11)) is more efficient than the total public or private financing (boundary solution), if the following condition is met:

\(^{18}\) The condition on higher interest rate for the private sector than the interest rate for the public sector may hold in the context of a partial equilibrium, when the public sector is characterized by lower risk. For a general equilibrium context see e.g. Feltenstein and Ha (1999).
\[ \theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < \min \{ I(q) \cdot r_{pr}, (I(q) + J(q)) \cdot r_{pu} \} \]  \hspace{1cm} (12)

The first part of condition (12), i.e., when \( \theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot I(q) \cdot r_{pu} < I(q) \cdot r_{pr} \) yields:

\[ \theta \cdot r_{pr} + (1-\theta) \cdot r_{pu} < r_{pr} \]  \hspace{1cm} (13)

\[ (1-\theta)(r_{pu} - r_{pr}) < 0 \]  \hspace{1cm} (14)

Inequality (14) holds for every \( \theta \in [e, 1) \).

The second part of condition (12):

\[ \theta \cdot r_{pr} + (1-\theta) \cdot r_{pu} - r_{pr} < \frac{J(q)}{I(q)} r_{pu} \]  \hspace{1cm} (15)

can be transformed to obtain the condition on \( \theta \):

\[ \theta < \frac{J(q)}{I(q)} \left( \frac{r_{pu}}{r_{pr} - r_{pu}} \right) \]  \hspace{1cm} (16)

Condition (16) implies that the capital share of the private partner in the partnership is determined by the percentage of savings achieved on the investment thanks to the private sector participation in the project and the interest rate spread of the private sector over the rate available to the public sector.

Condition (16) also allows to determine when the project should be executed by the private partner only. Setting the right hand side of the inequality to be larger or equal to unity we obtain \( J(q)/I(q) \geq (r_{pr} - r_{pu})/r_{pu} \). Therefore, the private partner should be the sole shareholder if the savings on the development outlays (in relation to \( I(q) \)) are higher than the relative spread between private and public rates.

\[ \text{Example 1:} \]

The relationship presented in condition (16) can be used to present the following stylized facts. One can assume \( J(q) \) in relation to \( I(q) \) to equal 20 percent.\(^{19}\) If private sector interest rates are assumed at 8.50 percent and an interest rate of long-term commercial loans for related government units at 7 percent, the PPP would be efficient in (in the sense of a tradeoff between cost and efficiency) when \( \theta < 0.2(0.07/0.015) \), i.e., when \( \theta < 93 \) percent and \( \theta \geq e \). Increasing the spread between the rates to 300 basis points yields a reduced private shareholding in the PPP to a maximum of 47 percent of total capital. Therefore, the bigger the difference between the interest rates for the public and private partners, the smaller the room for negotiation on capital participation between the parties.

\(^{19}\) The American Chamber of Commerce (2002), quoting British research (not providing the original source) claim that the savings from executing the investment by a private company equal 17 percent. If \( \frac{(I + J) - I}{I + J} = 0.17 \) then \( J/I = 0.17/0.83 \approx 20 \) percent.
Figure 1 shows the level of the fixed fee as a function of the public-private capital structure and the interval of efficient public-private financing for the case where $r_{pr}/r_{pu} > 1 + J(q)/I(q)$. Figure 1: Interval of efficient public-private financing (discrete model)

As can be seen from Figure 1, the level of the fixed fee $f(q, \theta)$ rises starting from the point where the project is realized only by the public sector ($\theta = 0$). The $f(q, \theta)$ increases as a result of the increase of the share of the more expensive private capital in the partnership. The rate of increase for ($\theta < e$) equals $I(q)(r_{pr} - r_{pu}) - J(q)r_{pu}$. At $\theta = e$ the transfer of knowledge occurs and the $f(q, \theta)$ drops by $(1 - e)Jr_{pu}$. For $\theta \geq e$ the $f(q, \theta)$ increases at the rate of $I(q)(r_{pr} - r_{pu})$. At $\theta_a$ the fixed fee $f(q, \theta)$ is equal to the fixed fee in a project without private sector participation; and at $\theta = 1$, with private-only shareholding, the fee equals $I(q)r_{pr}$.

Therefore, the larger the private share that is needed in the capital for the transfer of knowledge, the smaller the potential savings from private sector participation.

The transfer of know-how can also be described by a continuous function, where $\beta$ is any continuous and differentiable function of $\theta$. In the exemplary case of the linear function describing know-how transfer, function (9) can be written as:

$$f(q, \theta) = \theta \cdot I(q) \cdot r_{pu} + (1 - \theta) \cdot (I(q) + (1 - \theta) \cdot J(q)) \cdot r_{pu} \tag{17}$$

where $(1 - \theta) \cdot J(q)$ reflects the linear increase in outlays resulting from lack of know-how, proportional to the public partner share.

The first-order conditions for a minimum of function (17) with respect to $\theta$ are:

$$\frac{\partial f}{\partial \theta} = I(q) \cdot r_{pr} - (I(q) + 2(1 - \theta) \cdot J(q)) \cdot r_{pu} = 0 \tag{18}$$
Therefore \( f(q, \theta) \) has the minimum at such \( \theta^* \) that:

\[
\begin{align*}
(I(q) + 2(1-\theta^*) \cdot J(q)) \cdot r_{pu} &= I(q) \cdot r_{pr} \\
2(1-\theta^*) \cdot J(q) &= \frac{I(q) \cdot r_{pr}}{r_{pu}} - I(q) \\
\theta^* &= 1 - \frac{I(q)}{2J(q)} \left( \frac{r_{pr}}{r_{pu}} - 1 \right)
\end{align*}
\] (19) (20) (21)

Since the second derivative of (17) with respect to \( \theta \) is positive for each \( J(q) > 0 \), then equation (21) determines the minimum of function (17).

As \( \theta \) ranges from zero to one, the condition for \( \theta^* \) to be an interior minimum \((0 < \theta < 1)\) exists when \( r_{pr} - r_{pu} > 0 \) and \( \frac{r_{pr}}{r_{pu}} - 1 < \frac{2J(q)}{I(q)} \).

**Example 2:**

Equation (21) can be used to determine the private partner’s optimal shareholding for the same ratio of \( I/J = 5 \) as in Example 1. For the cases in which interest rates applicable to the private sector are on average 25 percent higher than the rates for the public sector \((r_{pr}/r_{pu} - 1 = 0.25)\), function \( f(q, \theta) \) reaches its minimum at \( \theta^* = 0.375 \). The optimal capital structure would then be a 62.5 percent share in capital owned by the public partner and 37.5 percent of the capital owned by the private partner.

PPP with mixed financing will be Pareto-efficient under these assumptions if the following condition is met:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot (I(q) + J(q)) \cdot r_{pu} < \min\left( I(q) \cdot r_{pr} ; (I(q) + J(q)) \cdot r_{pu} \right) \] (22)

For \( I(q) \cdot r_{pr} < (I(q) + J(q)) \cdot r_{pu} \) (first part of condition (22)), the private partner share \( \theta \) results from the solution of the condition:

\[
\theta \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot (I(q) + (1-\theta) \cdot J(q)) \cdot r_{pu} < I(q) \cdot r_{pr} \] (23)

\[
I(q) \cdot r_{pr} - (1-\theta) \cdot I(q) \cdot r_{pr} + (1-\theta) \cdot J(q) \cdot r_{pu} + (1-\theta)^2 \cdot J(q) \cdot r_{pu} < I(q) \cdot r_{pr} \] (24)

\[
(1-\theta)^2 \cdot J(q) \cdot r_{pu} < 0 \] (25)

This condition is met for \( 1 - \frac{I}{J} \left( \frac{r_{pr}}{r_{pu}} - 1 \right) < \theta < 1 \).
For \( I(q) \cdot r_{pr} > (I(q) + J(q)) \cdot r_{pu} \) (second part of condition (22)), the private partner share \( \theta \) must meet the following condition:

\[
I(q) \cdot r_{pr} - (1-\theta)I(q) \cdot r_{pr} + (1-\theta)J(q) \cdot r_{pu} + (1-\theta)^2 J(q) \cdot r_{pu} < (I(q) + J(q)) \cdot r_{pu} \quad (26)
\]

\[-I(q) \cdot (r_{pu} - r_{pr}) - J(q) \cdot r_{pu} + (1-\theta)I(q) \cdot (r_{pu} - r_{pr}) + (1-\theta)^2 J(q) \cdot r_{pu} < 0 \quad (27)\]

This condition is met for:

\[
0 < \theta < 2 - \frac{I}{J} \left( \frac{r_{pu}}{r_{pr}} - 1 \right) \quad (28)
\]

From the above analysis it turns out that for the section:

\[
\max \left[ 0; 1 - \frac{I}{J} \left( \frac{r_{pu}}{r_{pr}} - 1 \right) \right] < 0 < \min \left[ 2 - \frac{I}{J} \left( \frac{r_{pu}}{r_{pr}} - 1 \right); 1 \right] \quad (29)
\]

the PPP will be the efficient form of financing public investments.

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**Example 3:**

The level of expected savings of development outlays in existing joint venture partnerships can be calculated based on condition (29). In a sample of five public-private water supply and sewage companies in Poland,\(^{20}\) the level of \( \theta \) was between 0.33 and 0.64. For interest rates that on average are 20 percent higher for the private sector than for the public sector, and assuming that the existing capital structures are at an optimum, we can conclude that the ratio \( I/J \) was expected to be between 3.35 and 6.8, which amounts to savings of development outlays up to 12.8–23.0 percent. On average, these results are similar to the savings of 17 percent quoted in the literature for the U.K.

Figure 2 shows the space for Pareto-efficient public-private partnerships assuming a linear function describing the increase of costs due to the lack of know-how in the public sector.

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\(^{20}\) See Moszoro (2005), p. 70.
The transfer of knowledge begins even with a small private share in the shareholding. An efficient public-private capital structure is achieved in the interval \( \theta \in (0, \theta^*) \), where the fixed fee \( f(q, \theta) \) is lower than in the case of a sole public or sole private investment. The minimum value of the fixed fee is obtained at \( \theta^* \), where all the know-how is transferred from the private partner to the PPP. Further increases of \( \theta \) lead to an increasing \( f(q, \theta) \) as a result of the higher share of more expensive private capital. As in Figure 1, at \( \theta = 1 \) the fee equals \( I(q) \cdot r_{pr} \).

5. Conclusions

The model-based analysis in this paper shows that PPPs may provide public services cheaper than a sole private or sole public entity. Efficiency considerations suggest that the ownership of the SPV providing the public service does not have to be exclusively public or private. An optimum investment in public infrastructure requires mixed public and private ownership of the project and knowledge transfer. If the optimum share of private ownership \( \theta \) lies within the borders as defined by the interest rate spread and the potential savings from private management, a public-private capital structure will be more efficient in terms of lower fixed costs than a sole public or sole private ownership. Moreover, the larger the difference between the interest rates for the public and private partners and the smaller the savings resulting from private sector participation, the smaller the room for negotiation on capital participation between the parties.

This model-based conclusion has important policy implications. The economic motivations of the public and the private partners differ, which requires the legal framework of a PPP to be elaborate. From the point of view of the public partner, the transfer of knowledge that justifies the participation of the private partner in the SPV should be well
defined and secured in a properly drafted and executable legal documentation. From the point of view of the private partner, the lower cost derived from the public financing should be secured for the entire lifespan of the project. This might not be problematic if the funding of the project is provided upfront. However, if the funding is required over the lifetime of the project, the availability of cheaper financing would imply that the government involved maintains its creditworthiness and, accordingly, follows sound macroeconomic policies.

Therefore, PPPs may be most efficient in countries whose governments follow stability-oriented and predictable macroeconomic policies that are conducive to securing cheaper financing. An equally important advantage is a reliable legal system that provides the instruments to secure the interest of the public partner vis-à-vis the private partner. A lack of confidence between the partners, an insufficient legal framework, and the pursuit of other than stability-oriented macroeconomic policies would undermine the Pareto-efficient solution derived from the model. If any or all of these conditions are violated the possible savings achieved with the PPP scheme diminish.
Appendix. Composite Bond Rate Yields and Spreads

Table A-1. Corporate and Municipal Long-Term Bond Yields: 1990-2004

<table>
<thead>
<tr>
<th>Year</th>
<th>Prime Corporate Bonds, % Yield</th>
<th>High-Grade Municipal Bonds, Moody’s Aaa</th>
<th>C-M Bond Yields Spread</th>
<th>C/M Bond Yields Ratio</th>
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Period mean: 
Prime Corporate Bonds, % Yield: 7.31; 
High-Grade Municipal Bonds, Moody’s Aaa: 5.47; 
C-M Bond Yields Spread: 1.84; 
C/M Bond Yields Ratio: 1.33


Figure A-1: Corporate and Municipal Long-Term Bond Yields 1990-2004
Table A-2. Corporate, Municipal, and Treasury 10-year AAA-Bond Yields: October 2006-August 2009

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<th>Treasury Bonds</th>
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|            | Min  | 0.62 | 1.16 | 0.25 | 1.05 |
|            | Max  | 2.42 | 1.72 | 2.85 | 2.15 |
|            | Mean | 1.44 | 1.39 | 1.25 | 1.37 |
|            | Median| 1.34 | 1.36 | 1.13 | 1.29 |

Source: Reuters.
Figure A-2: Corporate, Municipal, and Treasury 10-year AAA-Bond Yields: October 2006-August 2009

Table A-3: US Treasury Bonds

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Note: the 20-year Treasury bond yields are calculated as an interpolation between the 10-year and 30-year bond yields.

Table A-6: Corporate-Municipal Bond Spread

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AAA bonds 0.94 0.85 0.99 1.40 1.05
AA bonds 1.20 1.16 1.31 1.38 1.26
A bonds 1.68 1.74 1.68 1.84 1.73

2yr 1.11 1.29 1.06 1.63 1.27
5yr 1.74 1.60 1.88 1.37 1.65
10yr 1.45 1.43 1.62 1.75 1.56
20yr 0.83 0.82 0.76 1.48 0.97
Table A-7: Corporate/Municipal Bond Spread Ratio

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</table>

Mean Spread | 1.48 | 1.47 | 1.50 | 1.58 | 1.51 |

AAA bonds | 1.31 | 1.26 | 1.32 | 1.43 | 1.33 |
AA bonds  | 1.56 | 1.54 | 1.60 | 1.58 | 1.57 |
A bonds   | 1.51 | 1.55 | 1.53 | 1.70 | 1.57 |

2yr       | 1.68 | 1.79 | 1.70 | 2.06 | 1.81 |
5yr       | 1.74 | 1.65 | 1.79 | 1.60 | 1.70 |
10yr      | 1.38 | 1.37 | 1.43 | 1.52 | 1.43 |
20yr      | 1.17 | 1.16 | 1.15 | 1.31 | 1.20 |

Figure A-3: Composite AAA-Bond Rates Spread

![Graph of Composite AAA-Bond Rates Spread](image-url)
Figure A-4: Composite AA-Bond Rates Spread

Figure A-5: Composite A-Bond Rates Spread
References


